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3 authors:

Anne-Marie Kietzig
McGill University

Savvas G Hatzikiriakos
University of British Columbia - Vancouver

Peter Englezos
University of British Columbia - Vancouver

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APPLIED PHYSICS REVIEWS—FOCUSED REVIEW

Physics of ice friction

Anne-Marie Kietzig, Savvas G. Hatzikiriakos,[a] and Peter Englezos
Department of Chemical and Biological Engineering, The University of British Columbia, 2360 East Mall, Vancouver, British Columbia V6T 1Z3, Canada

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Although the study of friction has a long history, ice friction has only been investigated during the last century. The basic physical concepts underlying the different friction regimes, such as boundary, mixed, and hydrodynamic friction are also relevant to ice friction. However, these friction regimes must be described with respect to the thickness of the lubricating liquidlike layer on ice. In this review the state of knowledge on the physics of ice friction is discussed. Surface melting theories are introduced. These theories attempt to explain the existence and nature of the liquidlike surface layer on ice at any temperature and without any load applied. Pressure melting, as the long-time explanation for the ease of ice friction, is discussed, together with the prevailing theory of frictional heating. The various laboratory setups for ice friction measurements are presented as well as their advantages and disadvantages. The individual influence of the different parameters on the coefficient of ice friction is discussed; these include the effects of temperature, sliding velocity, normal force exerted by the sliding object, the contact area between ice and slider, relative humidity, and also properties of the slider material such as surface roughness, surface structure, wettability, and thermal conductivity. Finally, the most important ice friction models based on the frictional heating theory are briefly introduced and research directions on the subject of ice friction are discussed. [doi:10.1063/1.3340792]

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I. INTRODUCTION

Even though friction on ice has only been investigated during the last century, the study of friction has a long history as shown in Fig. 1.

First encounters with frictional forces date back to the Neanderthal Age about 200 000 B.C. when people used frictional heat to make fire by rubbing wood on wood or striking flint stones. The first report of sliding on ice comes from Scandinavia around 7000 B.C. Rock carvings illustrate the use of a sledge for the transport of heavy goods. The next interesting historic record dates back to 2400 B.C. Egyptian carvings show that a lubricant, possibly water, was poured in front of a sledge to facilitate sliding.[1,2] The first recognition of the force of friction was found in Aristoteles' (384–322 B.C.) *Questiones Mechanicae*.[1] Another almost 2000 years passed before friction was studied for the first time quantitatively by Leonardo da Vinci (1452–1519). He investigated the influence of the apparent area of contact upon frictional resistance, distinguished between rolling and sliding friction, studied the benefits of lubricants, and made the first obser-

[a]Author to whom correspondence should be addressed. Electronic mail: hatzikir@interchange.ubc.ca.

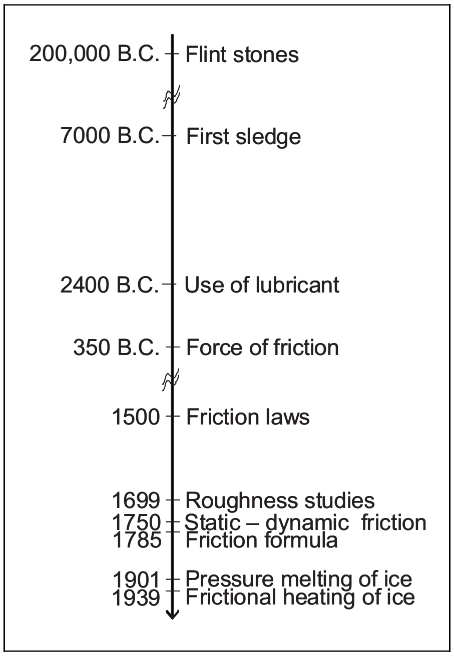

FIG. 1. Timeline of friction studies.

vations of wear. In fact, he postulated the two laws of friction.

1. *"Friction produces double the amount of effort if the weight be doubled"*—the force of friction is directly proportional to the applied load.
2. *"The friction made by the same weight will be of equal resistance at the beginning of its movement although the contact may be of different breadth and length"*—the force of friction is independent of the apparent area of contact for a given load.

However, since da Vinci's records were only published at the end of the 19th century, these laws are often referred to as Amonton's laws.[1,2] The French physicist Guillaume Amonton (1663–1705) rediscovered and thereby confirmed da Vinci's findings in 1699. Furthermore, he identified roughness as the fundamental cause of friction, and defined friction as the force required to lift interlocking asperities over each other during the sliding motion.[1] Leonard Euler (1707–1783) contributed to friction studies with a clear distinction of static and dynamic friction and introduced the symbol μ for the coefficient of friction.[1] In 1785, Charles Augustin Coulomb (1736–1806) investigated five main factors for frictional resistance. He studied the nature of materials in contact and surface coatings, the extent of the surface area, the normal pressure, the length of time that surfaces stay in contact, and the frictional behavior under vacuum as well as under varying ambient conditions namely temperature and humidity.

Coulomb was the first to formulate friction force as an equation

$$F_T = \mu F_N, \quad (1)$$

where F_T is the frictional force, F_N is the normal force, and μ is the coefficient of friction, which was assumed to be independent of the sliding velocity. This is sometimes referred to as the 3rd law of friction. However, it was found later that this only holds as long as the sliding velocity is not too low or not too high.[1,2]

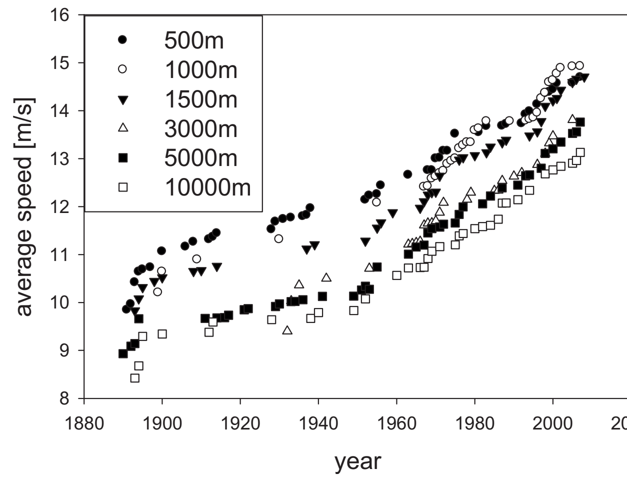

FIG. 2. Average speed of long track speed-skating world records (men) [data from ISU (Ref. 14)].

It was not until 150 years ago that ice became a matter of scientific investigations. Faraday brought two ice cubes into contact which instantly froze together.[3] He concluded that the ice surface is covered with a liquidlike layer. This marked the beginning of research efforts to understand ice, and the role its surface plays in ice friction. Shortly after Faraday's experiment, Thomson explained it by attributing the existence of the liquidlike layer to pressure melting.[4] Reynolds was the first to investigate systematically the matter of sliding on ice.[5] Following Thomson's ideas he wrongly attributed the low friction on ice to pressure melting; this explanation for the ease of skating was widely accepted among scientists for almost 40 years, until Bowden and Hughes[6] suggested that frictional heating might be the main contributor to the low friction coefficient on ice; this is today the generally accepted theory to explain ice friction.

The understanding of the underlying mechanisms of friction on ice is particularly important in a broad field of applications, such as motorized vehicle traffic in winter road conditions,[7–9] glacial movements, cargo transportation through northern sea ways, design of offshore structures and ice breakers,[10] and ice sports.[11] High friction on ice is desired for motorized vehicle traffic in winter road conditions and the grip of shoe soles on ice to avoid accidents. However, in the field of cargo transportation through northern sea ways and the design of offshore structures low friction materials are desired to limit maintenance and operation costs, e.g., 70% of the power of an ice breaker ship is consumed to overcome ice friction.[12] The reduction in ice friction is also important in competitive sports (de Koning, 1992; Rebsch, 1991).[13,14] This review mainly concentrates on the application in ice sports. The reason is that most of the published data in literature refer to such applications and the experimental setups address issues arising in ice sports. However, the knowledge gained can be applied to the other areas mentioned above. Another reason is that in competitive sports the urge for the faster speed is unbroken and keeps athletic competitions interesting.[15] As a result researchers around the world continuously search to find ways of minimizing ice friction. The development of world records in long track speed-skating over time is illustrated in Fig. 2. During speed-skating the contribution of ice friction to the overall frictional loss is about 20%.[13]

Colbeck[16] (1994) published an excellent review on snow friction. There are many similarities in snow and ice friction. Many experiments on snow friction are actually carried out on ice, whose behavior is less complex. The application of snow friction research lies mainly in the fields of snow sports and avalanche research. A review of the many different factors influencing ice friction and their interdependence with respect to different friction regimes clarifies our understanding of ice friction and sheds more light on the complexity of ice.

II. FRICTION REGIMES

Following, the basic physical concepts of dry, boundary, mixed, and hydrodynamic friction are introduced with respect to the thickness of the lubricating liquidlike layer on ice. The latter greatly influences the amount of friction on ice.

A. Dry friction

Dry friction describes the sliding contact of two surfaces in absence of any kind of lubricating layer. A solid surface is never completely flat but shows a distinct profile of surface asperities and valleys. When two asperities of different surfaces come into contact, adhesive bonds of chemical or physical nature are formed between these mating asperities. If the surfaces are moved relative to one another, the adhesive bonds are sheared. The force necessary to break the adhesive bonds between contacting asperities is the tangential friction force F_T given by

$$F_T = \tau_c A_c, \tag{2}$$

where τ_c is the shear strength, necessary to shear the asperity contact, and A_c is the area of real contact between the asperities of the mating surfaces.

Bowden[17] proposed that the real area of contact (A_c) between two surfaces is directly proportional to the applied load (F_N) and the softer material's hardness (H).

$$A_c = \frac{F_N}{H}. \tag{3}$$

Accordingly, the friction coefficient can be written as

$$\mu = \frac{\tau_c}{H}. \tag{4}$$

Thus, dry friction is characterized by the work necessary to break solid surface adhesive bonds. It depends on the applied load and the hardness of the surfaces but is independent of the sliding speed.[2,18,19]

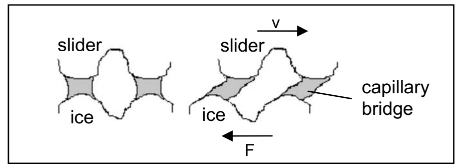

FIG. 3. Capillary bridges between asperities of contacting surfaces during sliding.

Real dry friction on ice under atmospheric conditions cannot exist. Even at very low temperatures a very thin liquidlike film lubricates the sliding interface. This film has a thickness of a few molecular layers.[20]

B. Boundary friction

Boundary lubrication is characterized by a lubricating layer with the thickness of only a few molecular layers between the sliding surfaces.[18] Boundary lubrication on ice is characterized by the temperature (T) in the contact zone being everywhere below the melting temperature (T_m), and the thickness of the lubricating liquidlike layer (h) being far smaller than surface roughness (R).[21]

$$\textit{Everywhere in contact zone}: T < T_m, \quad h \ll R.$$

The lubricating liquidlike layer reduces solid-solid contact between the surfaces. In total, the friction coefficient of boundary lubrication is typically lower than that of dry friction.[18]

C. Mixed friction

Mixed friction occurs when the surface temperature rises above the melting temperature (T_m) of ice at some points within the contact zone and the thickness of the liquidlike layer (h) is still less than the characteristic roughness of the surfaces (R).[21]

$$\textit{At some points in contact zone}: T > T_m, \quad h < R.$$

In this regime the load of the slider is partly supported by the surface asperities and partly by the lubricating layer. It is obvious that the increased thickness of the lubricating layer reduces solid-solid adhesion and enhances the lubrication.

Accordingly, the decrease in friction force compared to boundary lubrication can be demonstrated in

$$F_T = A_c * \left[\alpha\tau_c + (1-\alpha)\eta\frac{v}{h}\right], \tag{5}$$

where α is the fraction of unlubricated area, τ_c is the shear strength of the solid contact, v is the velocity of the slider, η is the viscosity, and h is the thickness of the lubricating layer.[18]

However, at the same time a wetting lubricant enforces the build-up of capillary water bridges between the asperities, as illustrated in Fig. 3.

The capillary bridges act as bonds between the slider and ice surfaces and exercise a drag force on the slider.[22] However, capillary bridges do not support the applied load. These capillary bridges act like liquid bonds and result in additional frictional resistance. Hence, capillary bridges should be

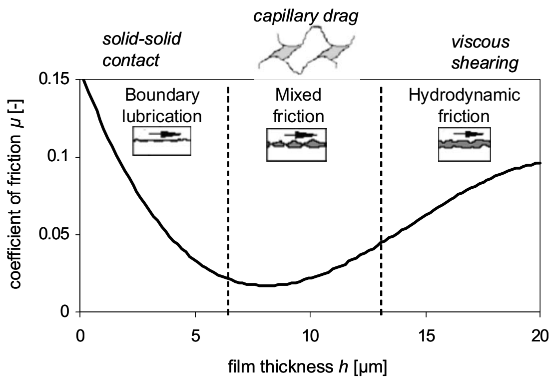

FIG. 4. Friction regimes relevant to ice friction depending on the thickness of lubricating layer [adapted from Bhushan (Ref. 18) and Colbeck (Ref. 22)].

taken into account to complement Eq. (5). The problem herein is, however, that no physical or experimental model exists to describe the contribution of capillary bridges to the friction force.

D. Hydrodynamic friction

If everywhere in the contact zone the temperature is above the melting temperature (T_m), and the thickness of the lubricating layer between the two surfaces is greater than the height of the asperities, friction is called hydrodynamic.[21]

$$\textit{Everywhere in contact zone}: T > T_m, \quad h > R.$$

In this friction regime the lubricating layer, not the surface asperities, carries the applied load. If the load is very high, a part of the lubricating layer might be squeezed out between the surfaces. However, for hydrodynamic friction, it is assumed that the thickness of the lubricating layer remains greater than the height of the asperities. Following the area of real contact is identical to the surface area (A) of the slider.[18] No solid-solid contact occurs during the sliding movement. Consequently, shearing of solid-solid adhesive bonds no longer contributes to the friction force. The frictional force can be described as

$$F_T = \tau_l A, \tag{6}$$

where τ_l is the shear strength of the lubricating liquidlike layer or an "effective" shear stress developed from shearing of the liquidlike layer. This can be simply expressed as

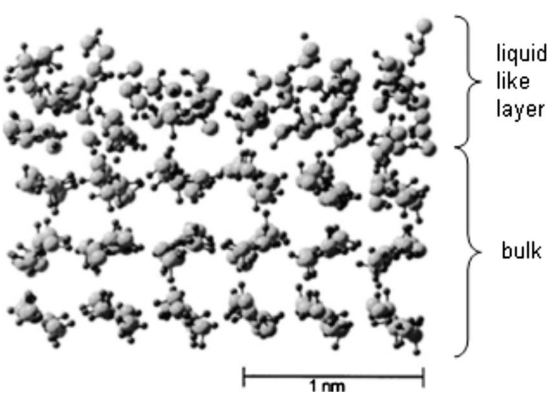

FIG. 5. Surface structure of ice (Ref. 24).

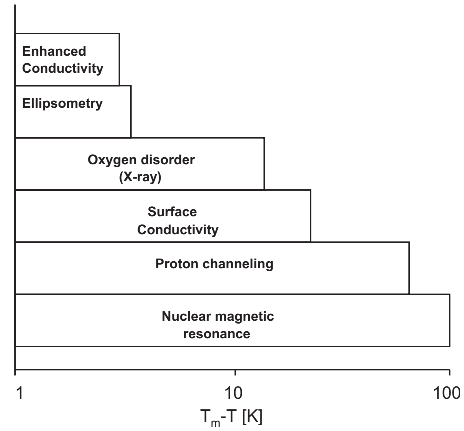

FIG. 6. Experimental techniques to investigate the liquidlike layer on ice [adapted from Petrenko and Whitworth (Ref. 20)].

$$\tau_l = \eta \frac{v}{h}. \tag{7}$$

As in the case for mixed friction capillary drag forces should be included in the case of water lubrication.

Fowler and Bejan[23] pointed out that the lubricating film under a slider on ice becomes thicker toward the trailing end. Consequently, friction mechanisms on ice can include all types of friction except for pure dry friction.

Figure 4 summarizes the various regimes of ice friction. Note the drop of the coefficient of friction with the film thickness in the boundary friction regime due to reduced solid-solid contact. On the other hand, the coefficient of friction increases with film thickness as it becomes fully hydrodynamic, as discussed above. Accordingly, there is an optimal film thickness associated with minimum friction for each slider system. It is also noted that there is a smooth transition between the different regimes indicated by the dashed lines.

III. ORIGIN OF THE LUBRICATING LIQUIDLIKE LAYER ON ICE

Ice sports are possible because of the existence of a liquidlike layer on the ice surface at temperatures below 0 °C. There are three different mechanisms that contribute to the thickness of the liquidlike layer. These are surface melting, pressure melting, and frictional heating.

A. Surface melting

Faraday[3] (1859) suggested the existence of a liquidlike layer as an inherent part of the ice surface. This layer exists even without the contact of another body, in other words no friction is required for its existence. At its surface the hexagonal structure of ice breaks down (Fig. 5). Experimental proof for the existence of the liquidlike layer was given with diverse experimental techniques. The temperature range in which the liquidlike layer was observed in different experiments shows a wide variability depending on the technique applied (Fig. 6). Therefore, it is not surprising that scientists came to different conclusions about the general nature of the liquidlike layer and more precisely the onset temperature of its formation. Different theories were developed to clarify the underlying physics. However, in spite of the vast experimental evidence, the scientific explanation for the presence of the liquidlike layer is still under debate. For a more detailed review the reader is referred to Petrenko and Whitworth.[20] Here some of the prevailing theories are summarized briefly.

Fletcher[25,26] attributed the formation of the liquidlike layer to *electrostatic interactions*, whereas Lacmann and Stranski[27] justified the existence of the liquidlike layer on nature's tendency to minimize the energy of a system. This theory of *free surface energy minimization* was further advanced by Dash *et al.*[28] They claimed that a system, whose free surface energy of a solid-vapor interface γ_{sv} is higher than the sum of the energies of the solid-liquid γ_{sl} and the liquid-vapor γ_{lv} interface, will *"lower its free energy by converting a layer of the solid to liquid"*

$$\gamma_{sv} > \gamma_{sl} + \gamma_{lv}. \tag{8}$$

While this theory holds for several solids, experimental evidence for ice shows that the free surface energy of "dry" ice is in fact not larger than the combined surface energies of the wetted system.[29–31]

Fukuta[32] suggested *subsurface pressure melting* as an explanation for the liquidlike layer on ice. Since water molecules at the ice surface only have water molecule neighbors from one side, they experience an inward pull. This pull exerts a pressure on the layers below. Makkonen[31] has shown that this pressure is large enough to reduce the melting point of the ice surface layer by about 13 K. It should be noted that even though subsurface pressure melting might contribute to the thickness of the liquidlike layer, it fails to explain its presence at lower temperatures.

Low-energy electron diffraction (LEED) experiments coupled with *molecular dynamics* simulation gave further insight into the forces leading to a liquidlike layer on ice. Surface molecules tend to vibrate and rotate constantly in order to minimize dangling bonds. Kroes[33] pointed out that through the movement of the molecules the outer ice layers become partially charged. Simulations of Devlin and Buch[34] confirmed that the breakdown of the solid structure is indeed energy minimizing, since thereby the number of dangling bonds can be reduced. Furthermore, they found that the surface is dynamically disordered. This was further explained by the results of Furukawa and Nada,[35] which indicated that molecules diffuse within the two outermost layers, which leads to this highly disordered surface. Finally, the study of Materer *et al.*[36] revealed that at a temperature as low as 90 K molecular vibration is so high that the outermost atoms could no longer be detected by LEED.

B. Pressure melting

For many years pressure melting was considered to be the explanation of choice for the low friction coefficient on ice. While it might contribute to the formation of a lubricating layer close to the melting point, pressure melting cannot explain the low friction on ice at lower temperatures. The pressure p exerted on the ice can be calculated by $p = F_N/A_c$. A major problem in friction studies in general is the calculation of the exact area of contact A_c. Since surfaces are never perfectly smooth, contact takes actually place between a certain number of asperities. With ice being a comparatively soft material the contact area of a slider on ice depends on the applied load. Furthermore, it also depends on the ambient temperature, because the ice softens with increasing temperatures reaching the melting point.

As an example consider an ice-skater (F_N=700 N), whose skate is in contact with the ice over an area of 1×10^{-4} m^2 (assuming a skate length of 0.1 m, width of 10^{-3} m, and only 10% of the geometric surface in contact with the ice, which might already be far too optimistic). Accordingly, the additional pressure here is 7 MPa. Based on the phase diagram of water dp/dT is approximately -13.5 MPa/K at 0 °C and 0.1 MPa. Therefore, a pressure increase of 7 MPa will only result in a temperature change of 0.5 K. Even with the herein assumed very low contact area pressure melting cannot contribute much to a liquid layer on ice, as the melting temperature of ice can only be lowered by 0.1 to 1 K depending on the slider. Colbeck[37] illustrated that only 0.005% of an ice skate is in actual contact with the ice for pressure melting and frictional heating to contribute equally to the heat production. This, however, would result in high wear rates and great losses in the film thickness by squeeze out of the lubricating layer, so that the friction coefficient would be higher than observed in experiments.

C. Frictional heating

On the basis of their experiments Bowden and Hughes[6] suggested that frictional heating plays a fundamental role in the low friction coefficient on ice. Heat generated by the frictional motion raises the temperature at the contacting points to the melting temperature of ice. Hence, the ice surface melts locally at the contacting asperities, whereby a noncontinuous melt water film is formed. This film contributes to the lubrication of the slider on ice. However, not all the energy from frictional heating is available for melting the ice due to energy consumption through material deformation and energy losses by heat conduction into the ice and the slider. Furthermore, the authors point out that this lubricating layer can become continuous, if the ambient temperature is close to the melting point of ice. They also note that the observed friction at 0 °C on wet snow is higher than on dry snow. However, any further conclusions regarding the drag effect of capillary bridges were not drawn in this study and as discussed previously their contribution to the coefficient of friction can be significant. In conclusion, it can be said that frictional heating is the most important contributor to the low friction on ice.

IV. EXPERIMENTAL METHODS FOR THE MEASUREMENT OF ICE FRICTION

Various experimental setups have been developed to measure friction on ice. Instrumented runners were used to measure real-life friction parameters;[38,39] rather big slider models were developed to have greater control over various parameters during field experiments.[40–43] However, to really understand ice friction at a fundamental level laboratory equipments proved to be most suitable.[6,12,21,44–58] In this section the various experimental setups, advantages, and disadvantages are summarized and discussed thoroughly.

A. Real-life experiments

Colbeck *et al.*[38] analyzed real-life ice skating with an instrumented skate. They have used thermocouples to assess the temperature at the skate surface and therewith gave experimental proof for the frictional heating theory. However, the friction coefficient could not be measured with this skate.

de Koning *et al.*[39] were first to attempt real-life analysis of ice skating with an instrumented skate. They developed ice skates which are instrumented with strain gages. These skates enabled the measurement of the push-off and friction force during skating. Experiments were made on different indoor and outdoor ice skating rinks with an experienced ice skater. The advantage of these experiments is that real-life skating conditions were analyzed. However, real-life experiments show limited control over the different variables. Different ice rinks use different ice making procedures and different locations imply different air temperatures, relative humidities, and water qualities, which will all have an effect on the measured friction coefficient. Furthermore, experiments with only one skater are not statistically significant and imply unpredictable variability. The results are greatly dependent on the skater's daily performance, i.e., his skating technique might not have been the same on different days at different locations, which results in different loads and pressure as well as in different velocities. Some of the friction results are presented below and compared with results obtained from other experimental setups.

B. Slider models

Bowden[40] used small sledges with rounded front edges cut from different materials to measure friction on snow and ice. He applied a certain procedure to prepare the ice surface. It is unclear how the sledges were accelerated.

Kuroiwa[41] reported ice friction measurements with real skate blades mounted to a frame. This slider model was ejected from a catapult to reach its sliding speed; thereby the velocity can be more closely monitored.

Slotfeldt-Ellingsen and Torgersen[42] and Itagaki *et al.*[43] also used automatically accelerated slider models of different size and weight to measure friction on ice. In contrast to Bowden[40] and Kuroiwa[41] they also applied an ice making procedure to further limit variability.

Friction measurements with slider models as described above reduce the unpredictable human factor in the experiments and increase the controllability of parameters such as the ice used, velocity, and load. However, other parameters, such as ice and air temperature, still contribute to variability in the measurements. One particular problem of slider models is that their sliding track cannot be completely predicted. Therefore, the slider model is likely to take different routes over the ice in each experiment. If this unpredictability is to be eliminated by using prepared tracks, the friction on the sidewalls of the track will contribute to the overall measured friction.

C. Linear experimental devices

Different linear devices were used by Jones *et al.*,[44] Montagnat and Schulson,[45] Ducret,[46] and Marmo.[47] These pieces of equipment vary in their setup but they all share the characteristic that the movement of the slider on the ice surface is guided by a control mechanism during the experiment. Accordingly, the problem of unpredictable sliding tracks is solved with linear experimental devices. Furthermore, the ice making procedure, the load, and velocity settings are all well controlled. Montagnat and Schulson[45] and also Ducret[46] also ensured a certain temperature setting by conducting the experiment in a cold room or freezer unit. Compared to other laboratory equipment, as will be explained further down, linear experimental devices have the advantage that friction between the slider and the ice can be investigated on a fresh ice surface.

D. Rotational experimental devices

Different kinds of rotational devices were used to measure friction on ice. Similar to linear experimental devices discussed above load and velocity can easily be set by the experimenter. One advantage of these setups is the rather small size of the equipment, which facilitates the use of cold boxes and temperature chambers. Accordingly, many setups include close surveillance of the ice and air temperature and even relative humidity. Furthermore, artificial ice surfaces can be created following an ice making procedure to limit variability.

Experiments with a rotary viscometer were carried out by Kozlov and Shugai.[21] This setup utilizes a flat metal ring that slides against a hollow cylinder. This setup necessitates measurements of the vertical displacement of the ring, since the rotation melts the ice at the cylinder wall.

Many experimenters made use of an ice ring or disk, which rotates against a stationary ice sample. Strausky *et al.*,[49] Buhl *et al.*,[50] Liang *et al.*,[51] and Kietzig *et al.*[52] successfully applied this setup for their analysis of ice friction. While these setups enable good controllable settings and limited variability, the sample continuously slides over the same ice. Accordingly, edge effects in front of the slider sliding over a fresh ice surface cannot be assessed as seen under real ice sport conditions or in other applications.

A setup with discontinuous ice-slider contact was applied by Evans *et al.*[48] and Petrenko,[12] who used a lathe setup with an ice cylinder rotating against a sample. Accordingly, not the whole circumference of the ice cylinder is in contact with the slider. Therefore, a particular area on the ice cylinder can refreeze before getting into contact with the slider again.

Another solution to the edge effect issue was shown by Bäurle,[53] who used a similar setup as described above with an ice turntable and a stationary sample mounted to an arm above it. The ice track with a diameter of 1.60 m is large enough to ensure that the ice surface refreezes before the next pass under the sample. Bowden and Hughes,[6] Oksanen and Keinonen,[54] and Lehtovaara[55] applied a variation in this setup with a rotating ice surface and the sample mounted to

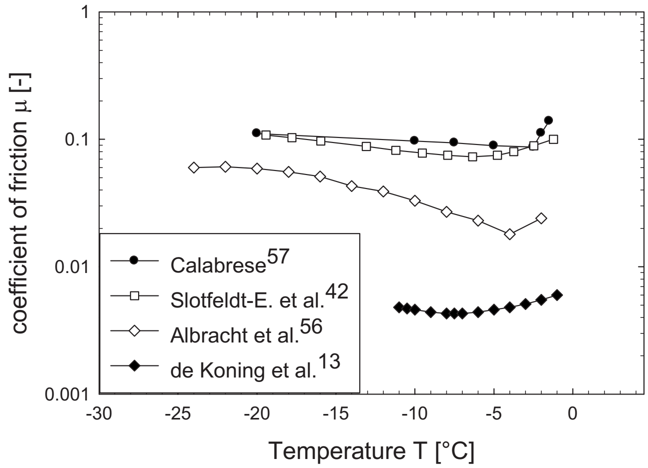

FIG. 7. Temperature dependence of the friction coefficient.

TABLE I. Experimental parameters for the data plotted in Fig. 7.

Reference	Slider material	F_N (N)	v (m/s)	A_c (mm^2)
Calabrese[a]	Steel ring (AISI 1018)	889.6	<1	1235
Slotfeldt-E. *et al.*[b]	HDPE slider block	100	0.3	15 000
Albracht *et al.*[c]	Cr-steel pin	1	0.13	≈2
de Koning *et al.*[d]	Steel skate blade ("Viking special")	700	8	≈400

[a]Reference 57.
[b]Reference 42.
[c]Reference 58.
[d]Reference 13.

an arm, which permits horizontal movement of the slider, as to ensure that the samples sees fresh ice after each turn.

Similarly, another variation in rotational experimental devices was used by Akkok *et al.*,[56] Calabrese,[57] and Albracht *et al.*[58] These experiments were carried out on a stationary ice surface with a rotating sample. Calabrese[57] used a ring; again there exists the problem with the continuous contact between the slider and the ice. Akkok *et al.*[56] used a ball on disk setup, which allows the ice to refreeze after the slider passes. Albracht *et al.*[58] even avoided sliding over the same track by using a spiral track for their pin slider.

In summary, it can be stated that in order to gain a better understanding of the different parameters that influence ice friction, laboratory devices such as those described above are necessary. They ensure good control over the settings such as ice and air temperature, humidity, velocity, and load. In addition the use of a certain ice making procedure further limits undesired variability in the results. On the other hand, small scale devices often have the characteristic of continuous ice-slider contact or a slider that slides over the same track. While this might not have an impact on the overall analysis and understanding of ice friction, it is rather different to what is seen in actual ice sports. To analyze the real overall ice sport performance in terms of race time, field experiments with athletes serve best. Hereby, a certain number of athletes have to be involved in the experiment to ensure statistical significance and eliminate variation resulting from the human factor. Studies comparing results obtained from real-life experiments involving humans with those from a laboratory setup are most welcome.

V. INFLUENCE OF DIFFERENT PARAMETERS ON THE FRICTION COEFFICIENT

Based on the information from ice friction experiments carried out by different researchers the influence of the different parameters on the friction coefficient will be discussed next in separate sections for each important parameter independently. In many cases, results from various sources are compared to identify consistency of experimental results in the literature. In some cases such comparisons were proven difficult as experimental findings were performed under different operating conditions.

A. Temperature *T*

Since the first ice friction studies by Bowden and Hughes,[6] many researchers have confirmed the dependence of the friction coefficient on temperature.[8,9,39,42,43,48,51,53,56–59] Some researchers merely report a decrease in the friction coefficient with increasing temperature.[8,9,43,48,51,53,56,59] This is the case when friction is dominated by boundary friction conditions. However, to obtain the full picture of the dependence of ice friction on temperature, it is necessary to consider all friction regimes, as introduced in Fig. 4.

Figure 7 depicts four sets of experimental results on the temperature dependence of the friction coefficient on ice across the whole range of friction regimes. Overall, the various data sets show the same trend, which has also been confirmed in snow friction studies.[40,50] The coefficient of friction decreases first with increasing temperature and rises again when the temperature approaches 0 °C. The minimum coefficient of friction is obtained between −2 and −7 °C depending on the method of measurement, the slider's normal load, the linear sliding speed, and the slider material. Obviously, at lower temperatures the friction is dominated by solid-solid interactions, typical for the ice friction curve to the left of the minimum, as illustrated in Fig. 4. At tempera-

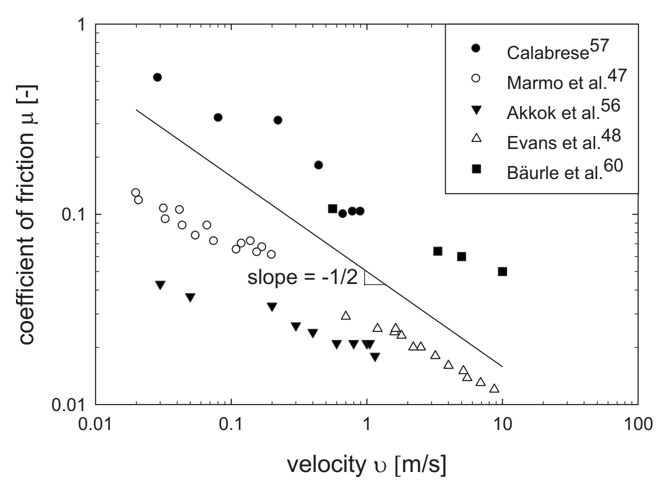

FIG. 8. Velocity dependence of the friction coefficient (enhanced lubrication).

TABLE II. Experimental parameters for the data sets plotted in Fig. 8.

Reference	Slider material	F_N (N)	T (°C)	A_c (mm^2)
Evans *et al.*[a]	Mild steel rod	45.4	−11.5	≈300
Marmo *et al.*[b]	Ice hemisphere over steel	2.1/2.4	−11.5	≈2
Akkok *et al.*[c]	Steel cylinder	75	−20	≈50
Calabrese[d]	Steel ring (AISI 1018)	889.6	−18	1235
Bäurle *et al.*[e]	PE block	84	−10	≈200

[a]Reference 48.
[b]Reference 47.
[c]Reference 56.
[d]Reference 57.
[e]Reference 60.

TABLE III. Experimental parameters for the data plotted in Fig. 9.

Reference	Slider material	F_N (N)	T (°C)	A_c (mm^2)
Jones *et al.*[a]	Formica block	196.2	≈0	15 000
Albracht *et al.*[b]	High alloy steel pin	1	−7	≈2
Bäurle *et al.*[c]	PE block	52	≈0	1000
de Koning *et al.*[d]	Steel skate blade ("Viking special")	706	−4.6	≈400

[a]Reference 44.
[b]Reference 56.
[c]Reference 60.
[d]Reference 13.

tures close to the melting point the thickness of the lubricating liquidlike layer becomes large enough; this not only facilitates the sliding but also adds to the resistance through the built-up of capillary bridges. The relevant friction regime is that of mixed friction beyond the minimum in the friction curve (Fig. 4) and with a further increase in film thickness hydrodynamic friction. However, the onset of this increase in friction depends largely on the slider material, the normal load, and the linear sliding velocity. Table I summarizes the differences in the operating parameters between the experiments and therewith provides explanation for the differences between the four sets of data. As it is discussed below the sliding velocity, the applied normal force, the area of contact, and the material of construction of the slider significantly influence the coefficient of friction.

B. Sliding velocity v

Bowden[40] first observed that friction against ice decreases with increasing velocity. Evans *et al.*,[48] who were first to model ice friction mathematically, confirmed these findings both experimentally and theoretically. Many other researchers found the same dependency using different experimental setups and materials.[41,45,47,56,57,60] Figure 8 illustrates some of these findings.

At higher velocities more frictional heat is produced than at slower speeds, resulting in a greater melt water production and therewith more lubrication, which facilitates the sliding motion; this is the case in the boundary friction regime and also the mixed friction regime before drag forces outweigh the benefits from a thicker lubricating layer (compare to Fig. 4). Overall, the five different sets of data shown here all agree with the predictions of theoretical models (presented below), which describe the velocity dependence in the boundary regime with $\mu \propto 1/\sqrt{v}$ [Ref. 48 (for high velocities), Ref. 54 (for friction dominated by thermal conductivity), and Refs. 56 and 61]. Many other sets of data not plotted here show a similarly decreasing trend in the coefficient of friction with increasing velocity. The differences among the various sets are attributed to the different experimental setups and operating conditions, such as slider material, normal force, temperature, and apparent contact area (see Table II for details).

Once drag forces considerably contribute to the overall friction force in the mixed friction and especially in the hydrodynamic friction regime the scaling of the friction coefficient with linear velocity changes dramatically. The coefficient of friction increases with velocity as first observed by Oksanen and Keinonen[54] in their ice against ice experiments at temperatures above −5 °C. They extended the mathematical model developed by Evans *et al.*[48] and found that the thickness of the melt water layer and therewith the coefficient of friction is proportional to $v^{1/2}$ for temperatures close

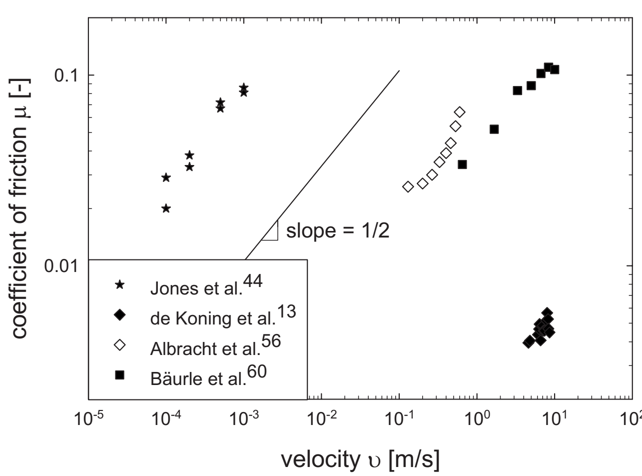

FIG. 9. Velocity dependence of the friction coefficient (added drag by melt water).

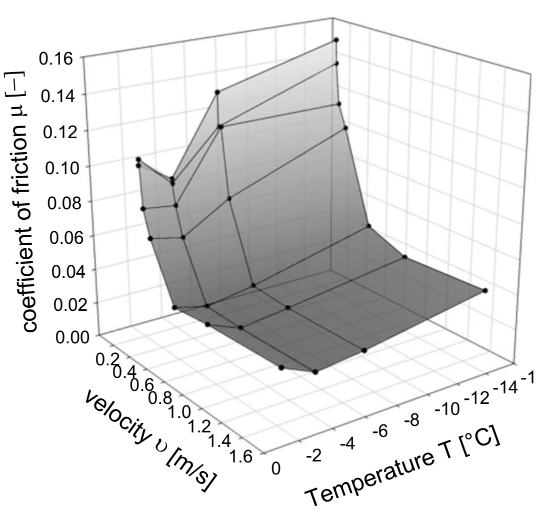

FIG. 10. Three-dimensional plot of temperature and velocity dependence of the friction coefficient (AISI 304L, F_N=3 N and A=147 mm^2) (Ref. 52).

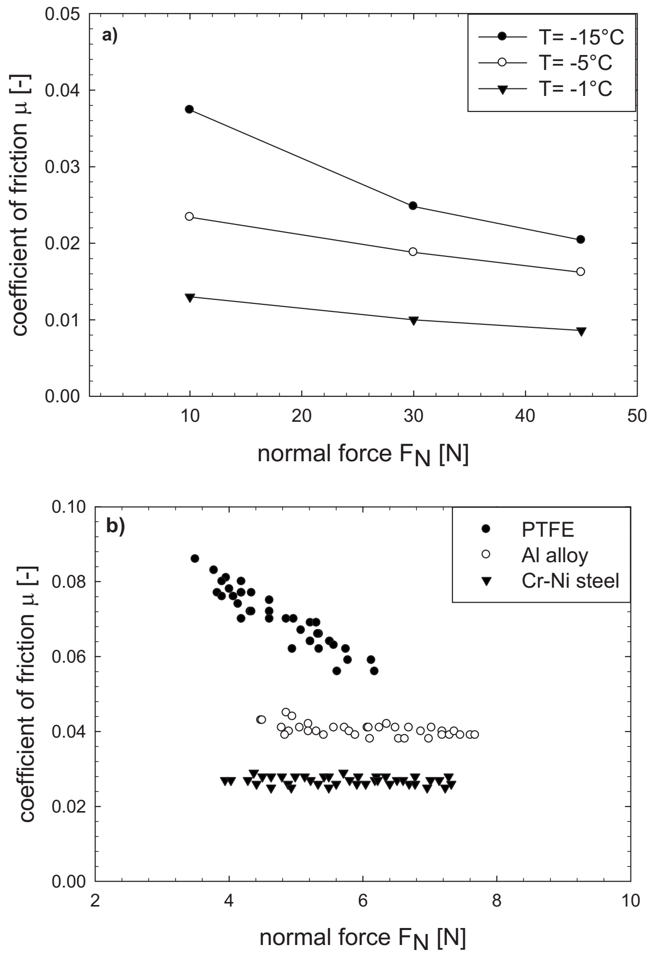

FIG. 11. Normal force dependence of the friction coefficient. (a) Data from Oksanen and Keinonen (Ref. 54); (b) data from Albracht *et al.* (Ref. 58).

to the melting point. Other researchers have confirmed these findings with different slider materials on ice as illustrated in Fig. 9.[39,44,58,60] The increase in friction with velocity can be explained through the increase in drag forces from shearing the lubricating layer in the hydrodynamic friction regime. The onset of this increase is obviously largely dependent on the size, weight, and material of construction of the slider, as well as the experimental temperature. Again differences in the experimental setup and operating conditions of the different sets of data are summarized in Table III.

In summary, it can be stated that by varying temperature or velocity widely all friction regimes from boundary to hydrodynamic can be identified in ice friction. Figure 10 shows the combined effect of temperature and velocity on ice friction. These results were found with a rotational experimental device as explained in Sec. V. A stainless steel ring AISI 304L was sliding at constant velocity against a polished ice surface at a set temperature and controlled normal force.[52] The friction coefficient initially decreases sharply with increasing velocity for all temperatures; however, for this particular experimental setup a slight increase in the friction coefficient could be noticed for velocities above 1 m/s, which can be attributed to added drag through capillary bridges, as explained above. For small velocities it can be clearly seen that friction is dominated by different mechanisms with increasing temperature. Furthermore, at around −4 °C a minimum in the friction coefficient can be observed. For temperatures below this minimum friction decreases with increasing temperature due to enhanced lubrication and reduced solid-solid contact. Close to the melting point additional resistance from capillary bridges and viscous shearing of the melt film increase friction again as discussed before.

TABLE IV. Experimental parameters for the data plotted in Figs. 11(a) and 11(b).

Reference	Slider material	v (m/s)	T (°C)	A_c (mm^2)
Oksanen and Keinonen[a]	Ice on ice	0.5	−15	11 475
			−5	
			−1	
Albracht *et al.*[b]	PTFE	1	−7	≈2
	Al alloy			
	Cr–Ni steel			

[a]Reference 54.
[b]Reference 58.

C. Normal force F_N

It is clear from the above discussion that the applied normal force and the apparent area of contact between the slider body and the ice surface play significant roles in the resulting coefficient of friction.

It is generally accepted in the literature that the friction coefficient of a slider against ice decreases with increasing normal force at a given temperature and velocity [Ref. 6 (ice-ice), Ref. 50 (PE-snow), Ref. 54 (ice-ice), and Refs. 56 and 58–60]. Derjaguin[59] has shown in his experiments, carried out with a steel slider, that for increasing loads at temperatures close to the melting point the friction coefficient becomes independent of the normal force. Similarly, Oksanen and Keinonen[54] have shown with their ice against ice experiments that at low temperatures (−15 °C) and slow sliding velocities (v=0.5 m/s) the coefficient of friction decreases with increasing normal force. However, at temperatures close to the melting point (−1 °C), the decrease is less pronounced [Fig. 11(a) and Table IV for experimental conditions]. Likewise, Calabrese's[57] experiments with a steel slider and loads above 400 N resulted in a coefficient of

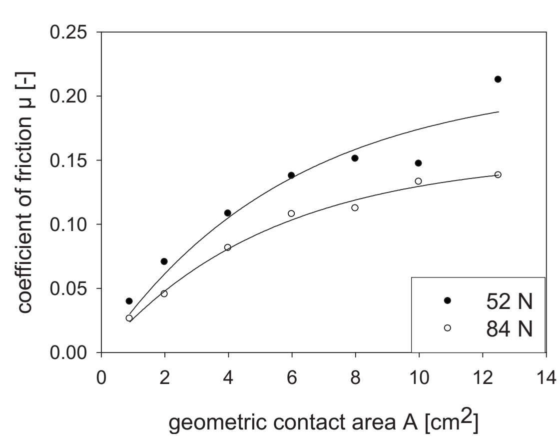

FIG. 12. Area and normal force dependence of the friction coefficient with T=−5 °C and v=3.3–5 m/s [data from Bäurle *et al.* (Ref. 60)].

friction, which is entirely independent of the applied load. Considering different slider materials, Akkok *et al.*[56] have shown that the coefficient of friction of a glass slider indeed decreases with normal force, while the corresponding results with a steel slider show no dependence. Similarly, Albracht *et al.*[58] also confirmed the decreasing trend of the coefficient of friction with normal force for a PTFE slider, while they have found that the friction coefficients of their aluminum alloy and chrome-nickel-steel sliders are independent of the applied normal force [Fig. 11(b) and Table IV for experimental conditions].

Interestingly, the independence of normal force was found for high load sliders made of high surface energy materials in experiments carried out at higher speeds and/or temperatures closer to the ice melting point. This indicates that these results refer to the hydrodynamic friction regime, where complete wetting of the slider dictates the results.

D. Apparent area of contact *A*

Bowden and Hughes[6] first performed experiments on the influence of the geometric area of the slider surface on the friction coefficient. Their experimental results have shown little dependence on the geometric contact area. However, Bäurle[62] has recently studied the influence of the geometric size of the slider under more controlled conditions. Figure 12 illustrates Bäurle's[60,62] experimental data with exponential curves fitted through the data points. The friction coefficient increases with increasing geometric contact area of the slider. The exponential growth curves show that the coefficient of friction tends to become independent of the applied force with larger contact areas. The initial sharp increase in the friction coefficient contradicts Leonardo da Vinci's second law of friction, which implies that the friction coefficient is independent of the apparent area of contact, and Bowden's[17] discussion of the influence of the area of contact on the coefficient of friction. Bäurle *et al.*[60] explain their results by the nature of the ice. For a very small geometric contact area the actual contacting asperities are located closer together, so that a larger amount of frictional energy is produced per unit area. This results into a thicker lubricating layer per unit area and an actual contact of close to 100%. The larger the geometric area of the slider, the more the contacting points are spread out. Hence, the dominating friction regime for a very small slider might already be hydrodynamic for a given temperature, velocity, and normal force setting, while the friction coefficient of a larger sample at the exact same experimental setting might still be controlled by asperity interactions. Furthermore, Fig. 12 illustrates again that the slider with the larger load shows lower friction, due to a greater amount of contacting points, which contribute to frictional heating and therewith to a thicker water layer. In conclusion, it is important to take the apparent contact area of different slider samples into consideration when comparing results of different researchers, as done above.

E. Roughness

In 1699, Amonton attributed friction to roughness.[1] This led to the assumption that smooth surfaces show less friction.

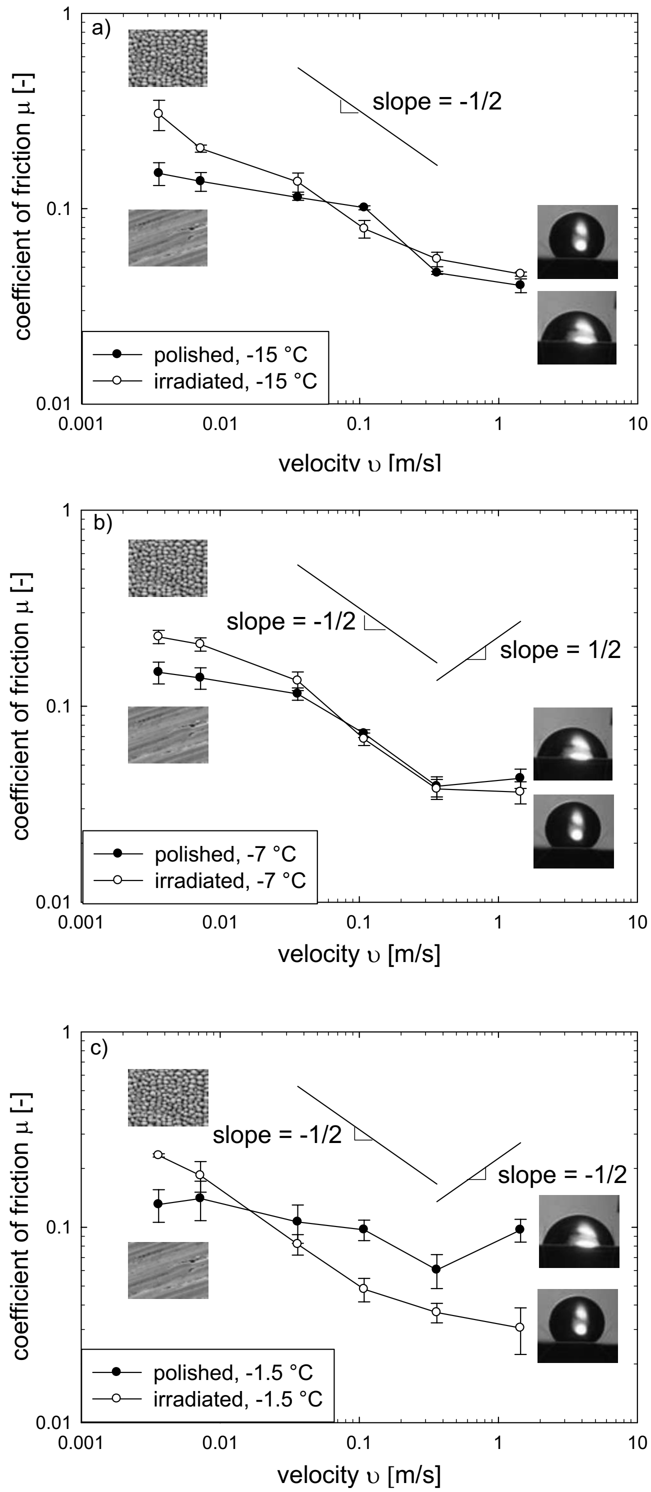

FIG. 13. Friction of hydrophilic, polished, and hydrophobic laser irradiated ASIS 304L sliders at (a) −15 °C, (b) −7 °C, and (c) −1.5 °C (Ref. 52).

Ice sports' athletes still follow this rule today. Calabrese[57] measured the friction coefficient as a function of sliding speed for steel with different degrees of roughness and confirmed that roughening increases the friction coefficient. Itagaki *et al.*[43] came to a similar conclusion after comparing different steel types with rough and smooth polish. Similarly, Ducret *et al.*[46] report that an increased roughness of the ice surface increases the friction coefficient of ultrahigh-molecular-weight-polyethylene sliding against ice at a very low sliding velocity of 2.5 mm/s. Generally, increasing the roughness of a surface leads to an increased surface area and more interlocking asperities during the sliding movement, which increases the wear rate and overall friction. Our latest results also confirmed this understanding (Fig. 13). Also, Marmo *et al.*[63] point out that roughness leads to a decreased actual thickness of the lubricating film. Melt water gets trapped in valleys between asperities, and a less uniform temperature profile exists due to hot spots at asperity tips. However, only few studies exist on the influence of roughness on ice friction. To the best of our knowledge there are no reports about the influence of roughness across all friction regimes. In the future more experiments should be done to thoroughly assess the influence of different quantifiable degrees of roughness on ice friction.

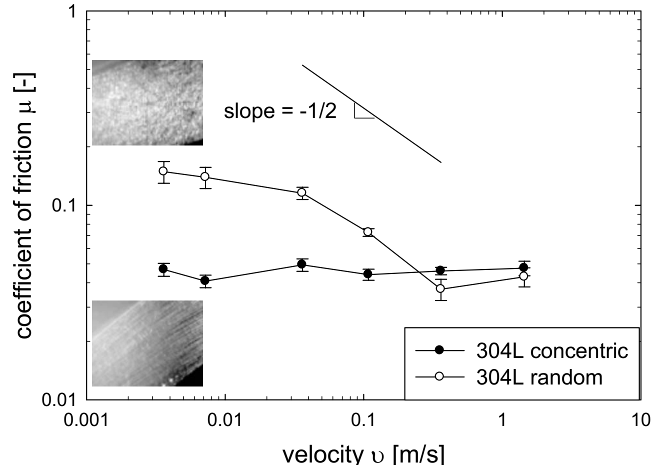

FIG. 14. AISI 304L slider with different surface structure at −7 °C (Ref. 52)

F. Wettability

Bowden[40] conducted experiments on the wettability of different materials and found that friction was highest for surfaces that wet easily, especially close to the melting point. This can be explained by the enhanced build-up of capillary bridges between the sliding surfaces, which becomes especially important in the mixed and hydrodynamic friction regime. However, it should be considered that a change in hydrophobicity was only achieved by using a different material. Therefore, the impact of hydrophobicity cannot be investigated independently of other material inherent parameters, such as thermal conductivity and material hardness. Furthermore, as was pointed out before, roughness and material surface structure play an important role, as well. However, no reference was made to the surface roughness of different materials. Colbeck[22] recognized the importance of capillary forces on snow friction. He has found that snow grains that do not support the slider's load directly are bonded to the slider surface by capillary bridges, whose formation is favored through increased melt water production with increasing sliding speed. Further investigations of the adhesion between water and static rough, hydrophilic and hydrophobic surfaces have shown that hydrophobicity clearly reduces the capillary bonding.[64,65]

Recently, we have conducted ice friction experiments with stainless steels sliders, which were initially hydrophilic but were rendered hydrophobic through femtosecond laser irradiation.[52,66] Without changing material parameters, such as thermal conductivity and hardness, this process presents a unique opportunity to investigate the influence of capillary drag on ice friction more closely. The laser irradiated slider surfaces exhibit a controlled dual-scale roughness, which is about twice as rough (R_a value) as the polished sliders'. Figure 13 shows the results from experiments with these two slider types. Due to the higher roughness the irradiated slider shows higher friction coefficients for very low sliding velocities and low temperatures, so to say in the boundary friction regime where asperity interaction dominates. For the experiments carried out at −1.5 °C the influence of surface wettability becomes most obvious [Fig. 13(c)]. While the coefficient of friction of the polished slider follows the typical trend with increasing speed discussed above (initial decrease, and after passing through a minimum the subsequent increase), the irradiated slider's coefficient of friction continuously decreases with increasing sliding speed. Accordingly, surface wettability has an important role in ice friction especially close to the melting point. Wettability being a material property and also a function of surface roughness is a significant factor to consider when conducting ice friction experiments.

G. Surface structure

Itagaki *et al.*[43] found in their experiments that their samples with polished grooves in sliding direction showed similarly low friction as highly polished sliders. In our recent work we examined the influence of surface structure more closely and found that for the same roughness value R_a the orientation of polishing marks plays a significant role on ice friction.[52] Experiments were conducted on a rotational experimental device as explained in Sec. V with two rings made from the same material, both polished to a R_a value of 600 nm. The difference in the two rings, however, is that one shows random polishing marks in all directions, while the other shows almost concentric grooves, which correspond

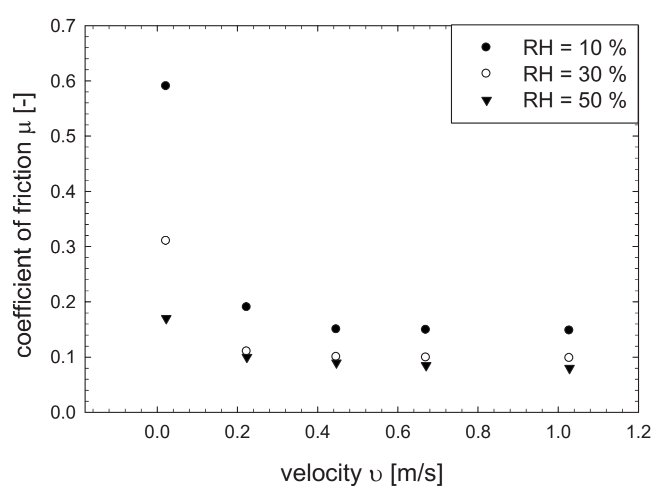

FIG. 15. The influence of relative humidity on the friction coefficient with F_N=177.9 N and T=−29 °C [data from Calabrese (Ref. 57)].

with the sliding direction. Sliders with random marks show the typical dependence of the coefficient of friction on velocity with $\mu \propto v^{-1/2}$, as also discussed before (Fig. 14). A minimum in the coefficient of friction can be observed. The coefficient of friction of the slider with the concentric grooves, however, stays almost constant and independent of sliding velocity. The far lower coefficient of friction at low velocities of the slider with grooves in sliding direction is explained by ice surface asperities running smoothly in these concentric grooves and thereby causing less interlocking than the random polishing marks of the other slider type.

With increasing speed, however, no significant changes are observed in the friction coefficient for the slider rings with concentric grooves. It seems like the decrease in friction from enhanced lubrication is immediately balanced off by added capillary drag of melt water being trapped in the grooves. Since our experiments are carried out with a ring, the melt water has no easy way to escape the concentric grooves. When the amount of melt water increases and can no longer be retained within the size of the grooves at the applied normal force, it is squeezed out. Accordingly, reduced asperity interaction through melt water and increased capillary drag seem to balance out each other with increasing velocity. On this basis it is understandable, why skis are commonly structured with V-shaped, diagonal, or linear grinding patterns, which serve to channel the lubricating melt water and thereby reduce capillary drag.[22]

H. Relative humidity RH

Calabrese[57] performed friction experiments with steel sliding against ice at different relative humidity conditions. His results are shown in Fig. 15.

Relative humidity has a strong influence on the onset of the sliding movement. The higher the humidity, the more lubricated is the sliding interface and the lower the frictional resistance. Unfortunately, no further experimental data exist for the influence of humidity at higher temperatures. Possibly at higher temperatures, where a thicker liquid layer exists, humidity is expected to have a minor effect on the coefficient of friction. However, this remains to be seen experimentally.

I. Thermal conductivity λ

Applying the frictional heating theory to ice friction, Bowden and Hughes[6] performed experiments on the influence of the slider's thermal conductivity on the friction coefficient. They compared the friction coefficient of a hollow ski with a copper surface to the one of the same ski construction filled with mercury. Air has a thermal conductivity of about 0.025 W/mK and mercury of 8 W/mK. The friction of the mercury filled ski was higher compared to the hollow air filled ski. Even though no further details are given about the exact experimental conditions, this result implies that the friction of a good thermal conductor is higher because less heat is available at the surface to melt the ice. Further experiments carried out by Itagaki *et al.*[43] with steels of different thermal conductivity have also shown this relationship between thermal conductivity and the friction coefficient. However, Albracht *et al.*[58] were not able to find a significant influence of thermal conductivity on ice friction in their experiments with different materials. Generally, as pointed out for wettability investigations, using different materials for the analysis of thermal conductivity cannot show the isolated effect of this parameter on ice friction. Using materials of different thermal conductivity always brings along a change in other material parameters such as wettability and material hardness, too. However, all analytical models on ice friction, as discussed below, include thermal conductivity as a major component in describing the melt water film thickness and therewith the resulting coefficient of friction.

VI. ICE FRICTION MODELS

From the above discussion of the isolated effect of single factors on ice friction it became clear that the different parameters interact significantly. This interdependence is one of the reasons that complicate the modeling of ice friction across the different friction regimes. Nonetheless, there has been a major effort to model ice friction and the most interesting of those models are briefly introduced here.

Evans *et al.*[48] developed the first theoretical explanation for the dependence of the friction coefficient on frictional heating. They defined the total frictional force F_T in terms of heat generated per unit displacement. Accordingly, the total frictional heat is described as the sum of three heat components.

$$F_T = F_S + F_I + F_M. \tag{9}$$

These are the heat conducted away from the interface through the slider material F_S, the amount of heat which diffuses into the ice F_I, and the heat that remains for melting the surface F_M. By assuming that the surface is at the melting temperature of ice and taking the length of contact from their experimental observations, this results into the following equation for the friction coefficient μ

$$\mu = \frac{A\lambda_S(T_m - T_0)}{F_N v} + \frac{B(T_m - T_0)}{F_N\sqrt{v}} + \mu_M, \tag{10}$$

with T_m and T_0 being the melting and the ambient temperature, respectively, λ_S being the thermal conductivity of the slider, v being the sliding velocity, A being a constant depending on the actual contact area, and B being a constant depending on the actual contact area, thermal conductivity and diffusivity of ice. The authors point out that it is not possible to calculate the contribution of the melt friction coefficient μ_M directly but an upper limit can be derived from experiments. Furthermore, from experimental and theoretical considerations they find the thickness of the lubricating layer to be smaller than the combined surface roughness, which indicates an overall mixed friction regime. Since the slider constantly moves over fresh ice, the temperature gradient ΔT_{ice} ($=|T_{\mathrm{contact}} - T_{\mathrm{ice}}|$; absolute value of the temperature difference between the contacting interface and the bulk ice) is generally greater than the one for the slider $\Delta T_{\mathrm{slider}}$ ($=|T_{\mathrm{slider}} - T_{\mathrm{contact}}|$; absolute value of the temperature difference between the bulk slider material and the contacting interface), except for the case of a very conductive slider material. Their experiments with copper, Perspex™, and steel indicate that 40% to 60% of the frictional heat are conducted away from the surface through the slider independent of the ambient temperature. The author's experiments have shown that the measured friction coefficient depends on the applied normal force according to $\mu \propto F_N^{-(1/3)}$. This disagreement with Eq. (10) results from the fact that the model [Eq. (10)] does not fully describe the dependence of the actual contact area on load.

The experimental dependency of the friction coefficient on velocity at −11.5 °C seemed different at higher and lower speeds. For low velocities the friction coefficient corresponds to $\mu \propto 1/v$ indicating that heat conduction into the slider F_S dominates, whereas for high velocities F_I takes over and $\mu \propto 1/\sqrt{v}$ explains the experimental results more closely. The results indicate that with increasing slider speed other mechanisms gain importance for friction on ice. However, other mechanisms, such as fluid mechanics of the squeezed lubricating film, were not included in the theory. Another interesting observation is that wear increased greatly above −2 °C with significant softening of the ice. The authors point out that wear will generally be higher on new fresh ice than observed in their studies, where a rod formed a track on an ice cylinder. However, the theory does not take energy losses by wear processes into account.

Oksanen and Keinonen[54] further elaborated this model. With the assumption that the lubricating layer is the main origin of frictional resistance, the authors combine the theory of Evans *et al.*[48] with hydrodynamic friction. Based on the assumption that the frictional motion results in a nonuniform heat transfer, they derive a model for the friction coefficient. The frictional heat Q_f generated by the motion during a certain time interval b/v is

$$Q_f = \mu F_N v \frac{b}{v}, \tag{11}$$

with b being the length of a contacting point. Equating this with the heat consumption equations from Evans *et al.*[48] results into

$$\mu = \frac{n^{1/4} A_c^{3/4}}{F_N} \left(\frac{1}{2} \frac{1}{(2v)^{1/2}} [\Delta T_I (\lambda_I c_I \rho_I)^{1/2} + \Delta T_S (\lambda_S c_S \rho_S)^{1/2}] + \left\{ \frac{1}{8v} [\Delta T_I (\lambda_I c_I \rho_I)^{1/2} + \Delta T_S (\lambda_S c_S \rho_S)^{1/2}]^2 + \eta_0 v h \rho_0 \right\}^{1/2} \right), \tag{12}$$

where n stands for the number of contacting points, A_c for the actual area of contact, λ_I and λ_S for the thermal conductivity, c_I and c_S for the specific heat capacity, and ρ_I and ρ_S for the density of ice or the slider material, respectively, ρ_0 for the density of and η_0 for the viscosity of water.

Here, the authors identify two regions with different relationships between the friction coefficient and the velocity. In the case of a great temperature gradient for the ice ΔT_{ice} thermal conductivity dominates over viscous shearing [the first part of Eq. (12)]. The same is true when the thickness of the lubricating layer is very small, suggesting that the produced heat is mainly conducted away and not available for melting the ice. The velocity dependency of the friction coefficient is then described by $\mu \propto 1/\sqrt{v}$ in the model. If the temperature gradients for the ice ΔT_{ice} and the slider $\Delta T_{\mathrm{slider}}$ are both small, which means that the ambient temperature is close to 0 °C, friction is governed by viscous shearing and melting of the ice [the second part of Eq. (12)]. The model gives a velocity dependency of $\mu \propto \sqrt{v}$ in this case. Furthermore, the authors point out that in the mixed region between the two cases discussed above the strength of the effects is determined by the velocity. At low velocities, thermal conductivity plays the greater role. At high velocities the time for heat conduction is reduced, resulting in more heat available for melting the ice. Even though the model indicates increased frictional resistance close to the melting temperature, it does not mention a potential contribution of capillary drag forces. Furthermore, energy losses due to wear mechanisms and squeeze out of the lubricating layer are ignored in the model. However, the proposed relationships between the sliding velocity and the resulting friction coefficient correspond reasonably well to experimental results, as shown in this paper's Sec. V B.

In contrast to the analytical models above, Akkok *et al.*[56] do not consider the melting temperature as an upper bound for the surface temperature but rather the softening temperature of ice. They argue that after reaching the softening temperature frictional motion wears the surface so much that the originally touching materials are no longer in contact. This is important, since through this kind of wear process no energy is consumed in the phase change but all heat is conducted away. The heat is assumed to be only conducted into the ice and not through the slider. The following model was developed,

$$\mu = C \frac{A_c (T_c - T_I)}{F_N} \left(\frac{\lambda_S \rho_S c_S}{v b} \right)^{1/2}, \tag{13}$$

where C is a constant, A_c the actual area of contact, T_c and T_I are the temperature at the contact and the ice, respectively. This model and further experiments confirm the results of Evans *et al.*[48] and Oksanen and Keinonen[54] with $\mu \propto 1/\sqrt{v}$. Furthermore, they emphasize that the velocity dependency changes at high temperatures, when hydrodynamic friction dominates. Concerning the dependency on load, their model gives $\mu \propto 1/\sqrt[4]{F_N}$ for partial contact, while the regression analysis on their experimental data gives $\mu \propto 1/\sqrt{F_N}$. The results of Evans *et al.*[48] lie in between these two. This variance indicates again that effects like the squeeze of the lubricating layer might play an important role here.

The analytical models so far recognize heat conduction and friction dominated by viscous shearing as two extreme cases in the determination of the overall frictional resistance. However, other important mechanisms are left on the side. Squeeze flow, for example, is not considered, which decreases the thickness of the lubricating layer at high loads and fast speeds.

Stiffler[61,67] included squeeze as a type of wear in his model. He derived the following model for the friction coefficient for a conducting surface:

$$\mu = \frac{2\lambda_S A_c (T_m - T_0)}{F_N(\pi\alpha_S l_c v)^{1/2}}, \qquad (14)$$

with α_S being the thermal diffusivity of the slider material and l_c the characteristic length of the contact. This analysis assumes hydrodynamic friction with a thickness of the lubricating layer that is larger than the combined roughness of both surfaces. He mentions that this assumption is unrealistic for an ice skater and concludes that the model is not suitable for this application.

Colbeck[22] developed a comprehensive model for snow friction, which recognizes the different mechanisms of friction acting at the same time

$$\mu = \mu_S + \frac{\mu_D \mu_W}{\mu_D + \mu_W}, \qquad (15)$$

with μ_S being friction due to capillary drag, μ_D being dry friction due to asperity interactions, and μ_W being wet friction due to shearing of the water film. The calculation of μ_W is based on the shear stress calculation for a Newtonian fluid with $\mu_W = c\mu_o v/h$, with μ_o being the water viscosity at 0 °C and c a constant that considers the area of contact but is based on the assumption that snow friction is independent of the applied load. The latter was found to not hold true for small sliders on ice as explained in this paper's Secs. V C and V D. Dry friction μ_D is based on the heat flow calculations, first introduced by Evans *et al.*[48] It is summarized in $\mu_D = \varepsilon e^{-\beta h}$, where β and ε are coefficients. While ε is not further defined, Colbeck[22] discusses four different cases of heat flow to derive at a value for β. These four cases are (1) heat flow into the ice only, (2) heat flow at contacts only, (3) entire lower surface at 0 °C, and (4) heat flow assuming an average temperature gradient. For an exact calculation for a particular slider information about the actual area of contact and the division of heat flow into slider and ice would be needed. μ_S is approximated by $\mu_S = \gamma h^3$, with γ being a constant. Even though this model is based on several assumptions regarding the constants, it gives a good example of the interaction of the different mechanisms controlling friction on snow and also ice.

Summarizing, only few analytical models for calculating the friction coefficient on ice exist. One problem with all these models is the estimation of the real contact area. An error can easily be introduced to the calculation, since the size and density of the surface asperities are unknown and have to be approximated. At the same time, adjusting the value for this variable is an easy way to adjust model results so that they fit experimental data. In general, the different models provide a good idea about the main effects interacting in frictional heating. However, other important mechanisms were not yet fully included in the theoretical analysis. The contribution of capillary drag forces to frictional resistance is not yet separated from overall hydrodynamic friction due to the lack of a sound physical understanding. The influence of material parameters such as surface wettability and roughness on drag force, squeeze flow, and the overall friction coefficient still need to be investigated in greater detail and included into a model.

VII. SUMMARY

Studies on ice friction impact not only ice sports but also other fields like glaciology, or ship hull design for cargo ship transportation through cold regions. While this review focuses on the application in ice sports, the findings are also relevant to other fields of application and thus contribute to their scientific understanding.

The nature of the liquidlike layer on ice is still a subject of scientific discussion. Surface melting is not yet fully understood and the ice as a material is still an interesting topic for research. In Sec. III it is pointed out that a minimum for friction on ice exists in the regime of mixed friction. This minimum was reported in terms of the optimal ice surface temperature for skating between −9 and −6 °C.[39,51] From the above discussion it is apparent that the location of this minimum depends on many factors. Based on the frictional heating theory, the influence of parameters like temperature, normal force, and velocity is fairly well understood. Different mathematical models explain ice friction depending on these parameters, as introduced in Sec. VI. These models consider heat conduction effects, hydrodynamic friction, and partly squeeze flow. However, friction regimes with a thinner lubricating layer are not well defined in models. Wear effects and capillary bridges between the slider and the ice were not yet successfully included into a model. Furthermore, the effect of material parameters such as roughness, hardness, and surface wettability still needs to be investigated in greater detail theoretically and experimentally, and their influence on the build-up of capillary bridges and the overall drag force has to be included into an ice friction model.

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